

# What can ultracold Fermi gases teach us about high $T_c$ superconductors and vice versa?

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## Abstract

We review recent developments in the field of ultracold atomic Fermi gases. As the cold atom system evolves from BCS to Bose-Einstein condensation (BEC), the behavior of the thermodynamics, and the particle density profiles evolves smoothly in a way which can be well understood theoretically. In the interesting “unitary” regime, we show that these and other data necessarily requires the introduction of a pseudogap in the fermionic spectrum which exhibits many striking similarities to its counterpart in underdoped high  $T_c$  superconductors. We emphasize these similarities, giving an overview of the experimental tools and key issues of common interest in both systems.

*Key words:* Ultracold Fermi gas, High  $T_c$  superconductors, Fermionic superfluidity, Feshbach resonance

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The study of ultracold trapped fermionic gases is a rapidly exploding subject [1,2] which is defining new directions in condensed matter and atomic physics. It has also captured the attention of physicists who study color superconducting quark matter as well as nuclear matter. Indeed, it is hard, in recent times, to find a subfield of physics which appeals this broadly to the research community. What makes these gases (and lattices) so important is their remarkable tunability and controllability. Using a Feshbach resonance, one can tune the attractive two-body interaction from weak to strong, and thereby make a smooth crossover from a BCS superfluid to a Bose-Einstein condensation (BEC) [3]. This allows high transition temperatures  $T_c$  (relative to the Fermi energy  $E_F$ ) which are interesting in their own right. Importantly, they may also provide insights into the high temperature cuprate superconductors [4,5,6].

This paper will concentrate on those issues relating to the common features of high  $T_c$  and cold atom systems with particular emphasis on the important “pseudogap” effects. We will study BCS-BEC crossover in atomic Fermi gases, looking at a number of different experiments which suggest the existence of noncondensed

pairs below  $T_c$  and, their counterpart, pre-formed pairs, above  $T_c$ . The latter are associated with the fermionic pseudogap. While there is much controversy in the field of high  $T_c$  superconductivity, there is a school of thought [5,6,7] which argues that these systems are somewhere intermediate between BCS and BEC. As stated by A. J. Leggett: “*The size of the [cuprate] pairs is somewhere in the range 10-30 Å – from measurements of the upper critical field, Fermi velocity and  $T_c$ . This means that the pair size is only moderately greater than the inter-conduction electron in-plane spacing, putting us in the intermediate regime of the so-called Bose-Einstein condensate to BCS superconductor (BEC-BCS) crossover, and leading us to expect very large effects of fluctuations (they are indeed found).*”

Studies of BCS-BEC crossover are built around early observations by Leggett [3] that the BCS ground state, proposed by Bardeen, Cooper, and Schrieffer in 1957

$$\Psi_0 = \Pi_{\mathbf{k}}(u_{\mathbf{k}} + v_{\mathbf{k}}c_{\mathbf{k},\uparrow}^\dagger c_{-\mathbf{k},\downarrow}^\dagger)|0\rangle, \quad (1)$$

is much more general than was originally thought. If one increases the strength of the attraction and self-consistently solves for the fermionic chemical potential

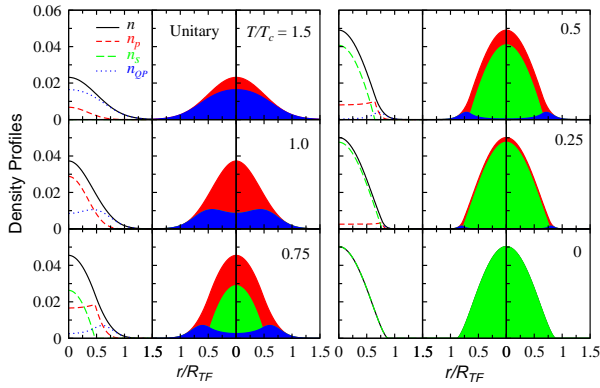


Fig. 1. (Color online) Decomposition of density profiles at various  $T$  at unitarity. Here green (light gray) refers to the condensate, red (dark gray) to the noncondensed pairs and blue (black) to the excited fermionic states.  $T_c = 0.27T_F$ , and  $R_{TF}$  is the Thomas-Fermi radius. Here  $n_s$ ,  $n_p$ , and  $n_{QP}$  denote density of superfluid, incoherent pairs, and fermionic quasiparticles, respectively.

$\mu$ , this wave function will correspond to a more BEC-like form of superfluidity. Knowing the ground state what is the nature of the superfluidity at finite  $T$ ? That is the central question we will address in this article.

Even without a detailed theoretical framework we can make three important observations. (i) The fundamental statistical entities in these superfluids are fermions. We measure the “bosonic” or pair-degrees of freedom indirectly via the fermionic gap parameter  $\Delta(T)$ . This tells us about bosons indirectly through the binding together of two fermions. (ii) As we go from BCS to BEC, pairs will begin to form at a temperature  $T^*$  above the transition temperature  $T_c$  at which they condense. This pair formation is associated with a normal state pseudogap. (iii) In general there will be two types of excitations in these BCS-BEC crossover systems. Importantly in the intermediate case (often called the “unitary” regime) the excitations consist of a mix of both fermions and bosons. They are not independent since the gap in the fermionic spectrum is related to the density of bosons.

To address  $T \neq 0$  we use a theoretical formalism based on a  $T$ -matrix approach which is outlined in detail elsewhere [5]. We turn now to experiments in cold atom systems which help to establish the existence of noncondensed pairs, and the related pseudogap. In Fig. 1 we plot a decomposition of the particle density profiles [8] in a trapped Fermi gas for various temperatures above and below  $T_c$ . The various color codes (or gray scales) indicate the condensate along with the noncondensed pairs and the fermions. This decomposition is based on the superfluid density so that all atoms participate in the condensation at  $T = 0$ .

This figure contains a key point. The noncondensed

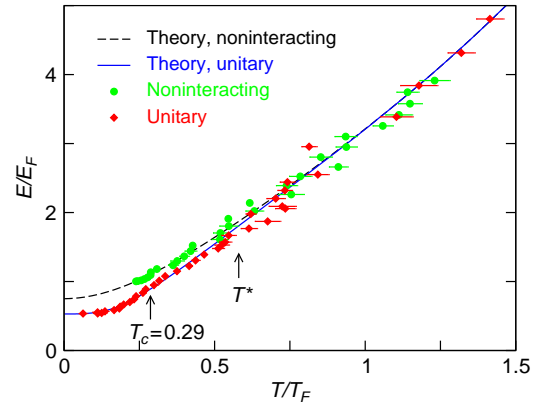


Fig. 2. (Color online) Pseudogap effects as apparent from thermodynamics. Plotted is energy per particle  $E/E_F$  versus  $T/T_F$ . From Ref. [9]. The fact that the experimental data (symbols) (and the two theoretical curves) for noninteracting and unitary Fermi gases do not merge until higher  $T^* > T_c$  is consistent with the presence of a normal state pseudogap.

pairs are responsible for smoothing out what otherwise would be a discontinuity between the fermionic and condensate contributions. This leads to a featureless profile, in agreement with experiment [2]. Indeed, these experimental observations originally presented a challenge for previous theories which ignored noncondensed pair excitations, and therefore predicted an effectively bimodal profile with a kink at the edge of the superfluid core. One can see from the figure that even at  $T_c$ , the system is different from a Fermi gas. That is, noncondensed pairs are present (in the central region of the trap) when the condensate is gone. Even at  $T/T_c = 1.5$  there is a considerable fraction of noncondensed pairs. It is not until around  $T^* \approx 2T_c$  for this unitary case, that noncondensed pairs have finally disappeared.

We next turn to a detailed comparison of theory and experiment for thermodynamics. Figure 2 presents a plot of energy per atom  $E$  as a function of  $T$  comparing the unitary and non-interacting regimes. The solid curves are theoretical while the data points are measured in  $^6\text{Li}$  [9]. There has been a recalibration of the experimental temperature scale [9] in order to plot theory and experiment in the same figure. The latter was determined via Thomas-Fermi fits to the density profiles [8]. To arrive at the calibration, we applied the same fits to the theoretically produced density profiles, examples of which appear in Fig. 1.

Good agreement between theory and experiment is apparent in Fig. 2. In the figure, the temperature dependence of  $E$  reflects primarily fermionic excitations at the edge of the trap [11], although there is a small bosonic contribution as well. Importantly one can see

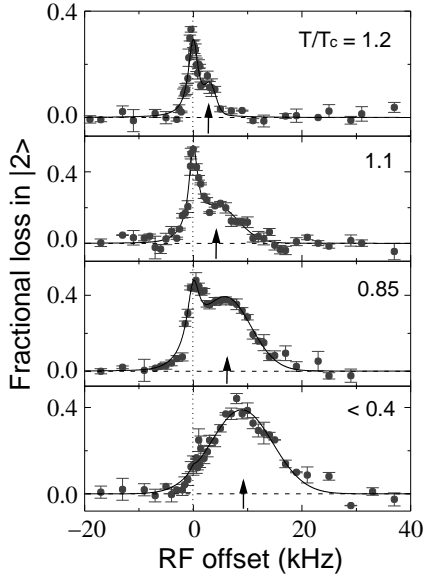


Fig. 3. Experimental RF Spectra for  ${}^6\text{Li}$  at unitarity. The temperatures labeled in the figure were computed theoretically at unitarity based on adiabatic sweeps from BEC. The two top curves, thus, correspond to the normal phase, thereby, indicating pseudogap effects. Here  $E_F = 2.5\mu\text{K}$ , or 52 kHz. From Ref. [10].

the effect of a pseudogap in the unitary case. That is, the temperature  $T^*$  is visible from the plots as that at which the non-interacting and unitary curves merge. This corresponds roughly to  $T^* \approx 2T_c$ . In this way, this figure and the proceeding figure are seen to be consistent.

Measurements [10] of the excitation gap  $\Delta$  can be made more directly, and, in this way one can further probe the existence of a pseudogap. This pairing gap spectroscopy is based on using a third atomic level, called  $|3\rangle$ , which does not participate in the superfluid pairing. Under application of radio frequency (RF) fields, one component of the Cooper pairs, called  $|2\rangle$ , is excited to state  $|3\rangle$ . If there is no gap  $\Delta$  then the energy it takes to excite  $|2\rangle$  to  $|3\rangle$  is the atomic level splitting  $\omega_{23}$ . In the presence of pairing (either above or below  $T_c$ ) an extra energy associated with the gap  $\Delta$  must be input to excite the state  $|2\rangle$ , as a result of the breaking of the pairs. Figure 3 shows a plot of the spectra for  ${}^6\text{Li}$  near unitarity for four different temperatures, which we discuss in more detail below. In general for this case, as well as for the BCS and BEC limits, there are two peak structures which appear in the data and in the theory [12,13]: the sharp peak at  $\omega_{23} \equiv 0$  which is associated with “free” fermions at the trap edge and the broader peak which reflects the presence of paired atoms; more precisely, this broad peak derives from the distribution of  $\Delta$  in the trap. At high  $T$  (compared to  $\Delta$ ), only the sharp feature is present, whereas at

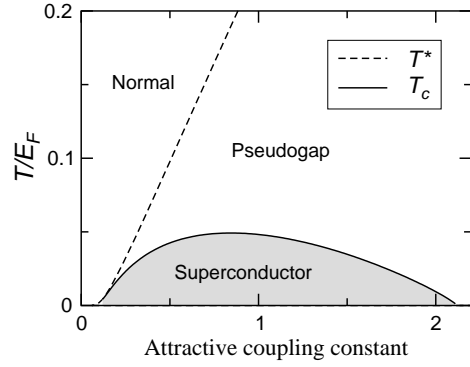


Fig. 4. Phase Diagram for a quasi-two-dimensional  $d$ -wave superconductor on a lattice. Here the horizontal axis corresponds to the strength of attractive interaction  $-U/4t$ , where  $t$  is the in-plane hopping matrix element.

low  $T$  only the broad feature remains. The sharpness of the free atom peak can be understood as coming from a large phase space contribution associated with the  $2 \rightarrow 3$  excitations [13]. These data alone do not directly indicate the presence of superfluidity; rather they provide strong evidence for pairing.

It is interesting to return to discuss the temperatures in the various panels. What is measured experimentally are temperatures  $T'$  which correspond to the temperature at the start of an adiabatic sweep from the BEC limit to unitarity. Here fits to the BEC-like profiles are used to deduce  $T'$  from the shape of the Gaussian tails in the trap. Based on knowledge [11] about thermodynamics (energy  $E$  in the previous figure or, equivalently, entropy  $S$ ), and given  $T'$ , one can then compute the final temperature in the unitary regime, assuming  $S$  is constant in a sweep. We find that the four temperatures are as indicated in the figures. Importantly, one can conclude that the first two cases correspond to a normal state, albeit not far above  $T_c$ . In this way, these figures suggest that a normal state pseudogap is present as reflected by the broad shoulder above the narrow free atom peak.

We turn finally to studies of the BCS-BEC crossover scenario in the high  $T_c$  superconductors. The cuprates are different from the ultracold fermionic superfluids in one key respect; they are  $d$ -wave superconductors and their electronic dispersion is associated with a quasi-two dimensional tight binding lattice. In many ways this is not a profound difference from the perspective of BCS-BEC crossover. Figure 4 shows a plot of the two important temperatures  $T_c$  and  $T^*$  as a function of increasing attractive coupling. On the left is BCS and the right is the intermediate or pseudogap (*i.e.*, near unitarity) regime. The BEC regime is not visible because  $T_c$  dis-

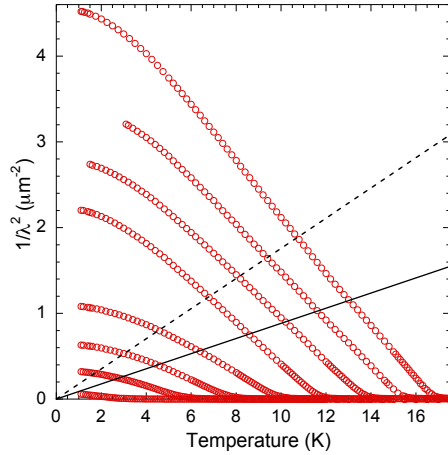


Fig. 5. Experimental data from Ref. [14] on superfluid density in the extreme underdoped regime of the cuprate superconductors.

appears before it can be accessed. This disappearance of  $T_c$  is a consequence of  $d$ -wave pairing which leads to pair localization effects [15]. If one chooses to fit the pairing temperature  $T^*(x)$  to experiment, this will relate the attractive coupling constant on the horizontal axis directly to the hole concentration  $x$ . The resulting figure [15] is then very similar to its counterpart in the cuprates.

To further probe the relevance of BCS-BEC crossover theory for the cuprates we now address data below  $T_c$  where noncondensed pairs are predicted to be present in addition to the usual fermionic Bogoliubov excitations. Figure 5 presents experimental data [14] on the inverse square (in-plane) penetration depth  $1/\lambda^2(T)$  for a series of very underdoped cuprates with variable hole stoichiometry or  $x$ . What is striking is the fact that the curves for each  $x$  are all rather similar even though the fermionic gap (or equivalently  $T^*$ ) varies considerably for this range of  $x$ . Thus, one can conclude that the behavior of the superfluid density  $n_s [\propto 1/\lambda^2(T)]$  does *not* simply reflect the fermionic gap energy scale. This observation, which would be paradoxical in strict BCS theory, is consistent with the picture that fermionic degrees of freedom are not the only excitations of the condensate to contribute to the superfluid density. Bosonic excitations are also present. Figure 6 presents a plot of the theoretical counterpart of  $1/\lambda^2(T)$  within the crossover scenario. Here there are bosonic as well as fermionic excitations of the condensate and the calculations appear to be in good semi-quantitative agreement with the data. The deviation from linearity at very low  $T$  is a consequence of incoherent pair excitations.

In conclusion, in this paper we have addressed commonalities of ultracold trapped Fermi gases and high

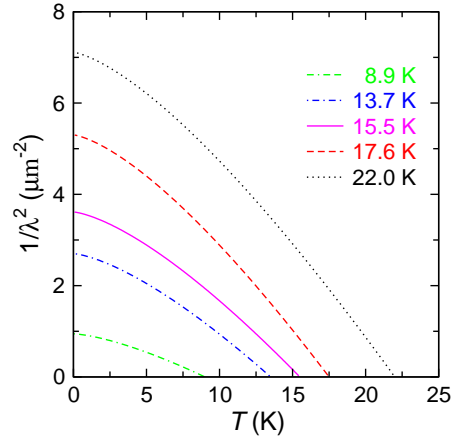


Fig. 6. Theoretical calculation of superfluid density for underdoped cuprate superconductors, to be compared with Fig. 5.

$T_c$  superconductors. These common features revolve around the scenario [6,5] that the cuprates are somewhere intermediate between BCS and BEC. Importantly, this scenario has been directly realized in trapped Fermi gases. Here one sees considerable evidence for pre-formed pairs and the related fermionic pseudogap which appear reminiscent of their cuprate counterparts.

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